8-1 Variation Functions

TEKS 2A.10.G Rational functions: use functions to model and make predictions in problem situations involving direct and inverse variation.

Objective
Solve problems involving direct, inverse, joint, and combined variation.

Vocabulary
direct variation
constant of variation
joint variation
inverse variation
combined variation

Also 2A.10.B, 2A.10.D

Why learn this?
You can use variation functions to determine how many people are needed to complete a task, such as building a home, in a given time. (See Example 5.)

In Chapter 2, you studied many types of linear functions. One special type of linear function is called *direct variation*. A **direct variation** is a relationship between two variables x and y that can be written in the form $y = kx$, where $k \neq 0$. In this relationship, k is the **constant of variation**. For the equation $y = kx$, y varies directly as x.

A direct variation equation is a linear equation in the form $y = mx + b$, where $b = 0$ and the constant of variation k is the slope. Because $b = 0$, the graph of a direct variation always passes through the origin.

EXAMPLE 1 Writing and Graphing Direct Variation

Given: y varies directly as x, and $y = 14$ when $x = 3.5$. Write and graph the direct variation function.

$y = kx$ — *y varies directly as x.*

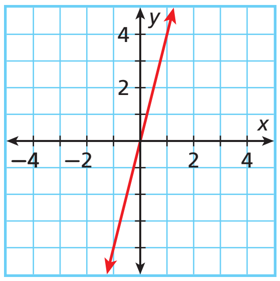
$14 = k(3.5)$ — *Substitute 14 for y and 3.5 for x.*

$4 = k$ — *Solve for the constant of variation k.*

$y = 4x$ — *Write the variation function by using the value of k.*

Graph the direct variation function.

The y-intercept is 0, and the slope is 4.



Check Substitute the original values of x and y into the equation.

$y = 4x$	
14	4(3.5)
14	14 ✔

Helpful Hint
If k is positive in a direct variation, the value of y increases as the value of x increases.

CHECK IT OUT!
1. Given: y varies directly as x, and $y = 6.5$ when $x = 13$. Write and graph the direct variation function.

When you want to find specific values in a direct variation problem, you can solve for k and then use substitution or you can use the proportion derived below.

$$y_1 = kx_1 \rightarrow \frac{y_1}{x_1} = k \quad \text{and} \quad y_2 = kx_2 \rightarrow \frac{y_2}{x_2} = k \quad \text{so,} \quad \frac{y_1}{x_1} = \frac{y_2}{x_2}.$$

EXAMPLE 2 Solving Direct Variation Problems

Geometry

Reading Math

The phrases "*y* varies directly as *x*" and "*y* is directly proportional to *x*" have the same meaning.

The circumference of a circle C varies directly as the radius r, and $C = 7\pi$ ft when $r = 3.5$ ft. Find r when $C = 4.5\pi$ ft.

Method 1 Find k.

$C = kr$

$7\pi = k(3.5)$ *Substitute.*

$2\pi = k$ *Solve for k.*

Write the variation function.

$C = (2\pi)r$ *Use 2π for k.*

$4.5\pi = (2\pi)r$ *Substitute 4.5π for C.*

$2.25 = r$ *Solve for r.*

Method 2 Use a proportion.

$\frac{C_1}{r_1} = \frac{C_2}{r_2}$

$\frac{7\pi}{3.5} = \frac{4.5\pi}{r}$ *Substitute.*

$7\pi r = 15.75\pi$ *Find the cross products.*

$r = 2.25$ *Solve for r.*

The radius r is 2.25 ft.

CHECK IT OUT!

2. The perimeter P of a regular dodecagon varies directly as the side length s, and $P = 18$ in. when $s = 1.5$ in. Find s when $P = 75$ in.

A **joint variation** is a relationship among three variables that can be written in the form $y = kxz$, where k is the constant of variation. For the equation $y = kxz$, y varies jointly as x and z.

EXAMPLE 3 Solving Joint Variation Problems

Geometry

The area A of a triangle varies jointly as the base b and the height h, and $A = 12\ \text{m}^2$ when $b = 6$ m and $h = 4$ m. Find b when $A = 36\ \text{m}^2$ and $h = 8$ m.

Step 1 Find k.

$A = kbh$ *Joint variation*

$12 = k(6)(4)$ *Substitute.*

$\frac{1}{2} = k$ *Solve for k.*

Step 2 Use the variation function.

$A = \frac{1}{2}bh$ *Use $\frac{1}{2}$ for k.*

$36 = \frac{1}{2}b(8)$ *Substitute.*

$9 = b$ *Solve for b.*

The base b is 9 m.

CHECK IT OUT!

3. The lateral surface area L of a cone varies jointly as the base radius r and the slant height ℓ, and $L = 63\pi\ \text{m}^2$ when $r = 3.5$ m and $\ell = 18$ m. Find r to the nearest tenth when $L = 8\pi\ \text{m}^2$ and $\ell = 5$ m.

A third type of variation describes a situation in which one quantity increases and the other decreases. For example, the table shows that the time needed to drive 600 miles decreases as speed increases.

Speed (mi/h)	Time (h)	Distance (mi)
30	20	600
40	15	600
50	12	600

This type of variation is an inverse variation. An **inverse variation** is a relationship between two variables x and y that can be written in the form $y = \frac{k}{x}$, where $k \neq 0$. For the equation $y = \frac{k}{x}$, y varies inversely as x.

EXAMPLE 4 Writing and Graphing Inverse Variation

Given: y varies inversely as x, and $y = 3$ when $x = 8$. Write and graph the inverse variation function.

$y = \frac{k}{x}$ — *y varies inversely as x.*

$3 = \frac{k}{8}$ — *Substitute 3 for y and 8 for x.*

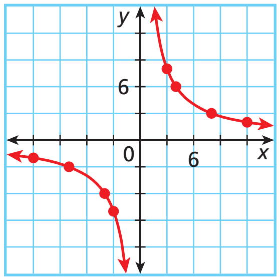
$k = 24$ — *Solve for k.*

$y = \frac{24}{x}$ — *Write the variation function.*

To graph, make a table of values for both positive and negative values of x. Plot the points, and connect them with two smooth curves. Because division by 0 is undefined, the function is undefined when $x = 0$.

x	y
−3	−8
−4	−6
−8	−3
−12	−2

x	y
3	8
4	6
8	3
12	2



Helpful Hint

When graphing an inverse variation function, use values of x that are factors of k so that the y-values will be integers.

CHECK IT OUT! 4. Given: y varies inversely as x, and $y = 4$ when $x = 10$. Write and graph the inverse variation function.

When you want to find specific values in an inverse variation problem, you can solve for k and then use substitution or you can use the equation derived below.

$$y_1 = \frac{k}{x_1} \rightarrow y_1x_1 = k \quad \text{and} \quad y_2 = \frac{k}{x_2} \rightarrow y_2x_2 = k \quad \text{so,} \quad y_1x_1 = y_2x_2.$$

EXAMPLE 5 *Community Service Application*

The time t that it takes for a group of volunteers to construct a house varies inversely as the number of volunteers v. If 20 volunteers can build a house in 62.5 working hours, how many volunteers would be needed to build a house in 50 working hours?

Method 1 Find k.

$t = \frac{k}{v}$

$62.5 = \frac{k}{20}$ — *Substitute.*

$1250 = k$ — *Solve for k.*

$t = \frac{1250}{v}$ — *Use 1250 for k.*

$50 = \frac{1250}{v}$ — *Substitute 50 for t.*

$v = 25$ — *Solve for v.*

Method 2 Use $t_1v_1 = t_2v_2$.

$t_1v_1 = t_2v_2$

$62.5(20) = 50v$ — *Substitute.*

$1250 = 50v$ — *Simplify.*

$25 = v$ — *Solve for v.*

So 25 volunteers would be needed to build a home in 50 working hours.

CHECK IT OUT! 5. **What if...?** How many working hours would it take 15 volunteers to build a house?

You can use algebra to rewrite variation functions in terms of k.

Direct Variation

$$y = kx \rightarrow k = \frac{y}{x}$$

Constant ratio

Inverse Variation

$$y = \frac{k}{x} \rightarrow k = xy$$

Constant product

Notice that in direct variation, the *ratio* of the two quantities is constant. In inverse variation, the *product* of the two quantities is constant.

EXAMPLE 6 Identifying Direct and Inverse Variation

Determine whether each data set represents a direct variation, an inverse variation, or neither.

A

x	3	8	10
y	9	24	30

In each case, $\frac{y}{x} = 3$. The ratio is constant, so this represents a direct variation.

B

x	4.5	12	2
y	8	3	18

In each case, $xy = 36$. The product is constant, so this represents an inverse variation.

CHECK IT OUT! **Determine whether each data set represents a direct variation, an inverse variation, or neither.**

6a.

x	3.75	15	5
y	12	3	9

6b.

x	1	40	26
y	0.2	8	5.2

A **combined variation** is a relationship that contains both direct and inverse variation. Quantities that vary directly appear in the numerator, and quantities that vary inversely appear in the denominator.

EXAMPLE 7 *Chemistry Application*

The volume V of a gas varies inversely as the pressure P and directly as the temperature T. A certain gas has a volume of 10 liters (L), a temperature of 300 kelvins (K), and a pressure of 1.5 atmospheres (atm). If the gas is compressed to a volume of 7.5 L and is heated to 350 K, what will the new pressure be?

Helpful Hint

A kelvin (K) is a unit of temperature that is often used by chemists.

0°C = 273.15 K

100°C = 373.15 K

Step 1 Find k.

$V = \frac{kT}{P}$ *Combined variation*

$10 = \frac{k(300)}{1.5}$ *Substitute.*

$0.05 = k$ *Solve for k.*

Step 2 Use the variation function.

$V = \frac{0.05T}{P}$ *Use 0.05 for k.*

$7.5 = \frac{0.05(350)}{P}$ *Substitute.*

$P = 2.\overline{3}$ *Solve for P.*

The new pressure will be $2.\overline{3}$, or $2\frac{1}{3}$, atm.

CHECK IT OUT! **7.** If the gas is heated to 400 K and has a pressure of 1 atm, what is its volume?